Fi8000
Valuation of Financial Assets

Spring Semester 2010

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Derivatives - Overview

Derivative securities are financial contracts that derive their value from other securities.

They are also called **contingent claims** because their payoffs are contingent on the prices of other securities.

Derivatives - Overview

- ☺ Examples of underlying assets:
  - Common stock and stock index
  - Foreign exchange rate and interest rate
  - Agricultural commodities and precious metals
  - Futures
- ☺ Examples of derivative securities:
  - Options (Call, Put)
  - Forward and Futures
  - Fixed income and foreign exchange instruments such as swaps

Derivatives - Overview

- ☺ Trading venues:
  - Exchanges – standardized contracts
  - Over the Counter (OTC) – custom-tailored contracts
- ☺ Serve as investment vehicles for both:
  - Hedgers (decrease the risk level of the portfolio)
  - Speculators (increase the risk)

A Call Option

A European* call option gives the buyer of the option ***a right to purchase*** the underlying asset, at the contracted price (the ***exercise*** or ***strike price***) on a contracted future date (***expiration***)

*An ***American*** call option gives the buyer of the option (long call) a right to buy the underlying asset, at the exercise price, ***on or before*** the expiration date

Call Option - an Example

A March (European) call option on Microsoft stock with a strike price of $20, entitles the owner with a right to purchase the stock for $20 on the expiration date*.

What is the owner's payoff on the expiration date? What is his profit if the call price is $7? Under what circumstances does he benefit from the position?

* Note that exchange traded options expire on the third Friday of the expiration month.

The Payoff of a Call Option

- On the expiration date:
  - If Microsoft stock had fallen below $20, the call would have been left to expire worthless.
  - If Microsoft was selling above $20, the call owner would have found it optimal to exercise.
- Exercise of the call is optimal if the stock price exceeds the exercise price:
  - Payoff at expiration is the maximum of two:

    Max {Stock price – Exercise price, 0} = Max {S_T – X, 0}

  - Profit at expiration = Payoff at expiration - Premium

Notation

S = the price of the underlying asset (**S**tock) (we will refer to S_0, S_t or S_T)

C = the price of a **C**all option (premium) (we will refer to C_0, C_t or C_T)

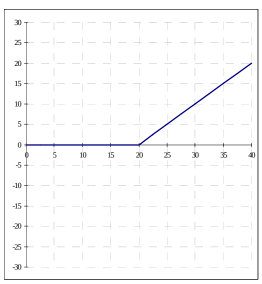
X or K = the e**X**ercise or stri**K**e price

T = the expiration date

t = a time index

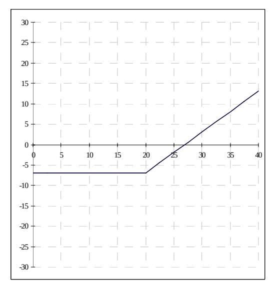
Buying a Call – Payoff Diagram

Stock price = S_T	Payoff = Max{S_T-X, 0}
0	0
5	0
10	0
15	0
20	0
25	5
30	10

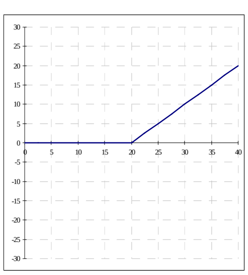


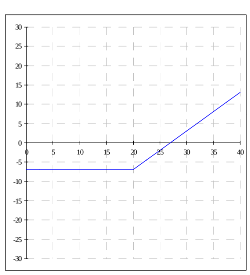
Buying a Call – Profit Diagram

Stock price = S_T	Profit = Max{S_T-X,0}-C
0	-7
10	-7
20	-7
25	-2
30	3
35	8
40	13



Buying a Call
Payoff and Profit Diagrams





Writing a Call Option

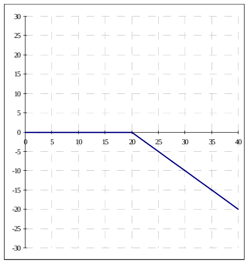
The seller of a call option is said to **write a call**, and he receives the option price called a **premium**. He **is obligated** to deliver the underlying asset on expiration date (European), for the exercise price.

The payoff of a short call position (writing a call) is the negative of long call (buying a call):

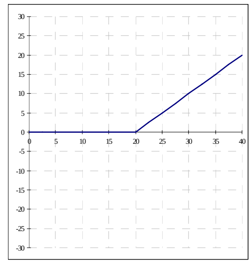
-Max {Stock price – Exercise price, 0} = -Max {S_T – X, 0}

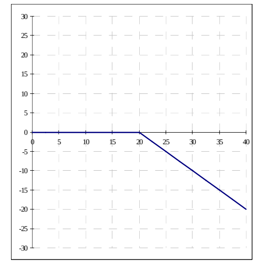
Writing a Call – Payoff Diagram

Stock price $= S_T$	Payoff = $-Max\{S_T-X,0\}$
0	0
10	0
15	0
20	0
25	-5
30	-10
40	-20



Buying a Call vs. Writing a Call Payoff Diagrams





Moneyness

- We say that an option is ***in-the-money*** if the payoff from exercising is positive
  - A call options is in-to-money if $(S_t – X) > 0$ (i.e. if stock price > strike price)
- We say that an option is ***out-of-the-money*** if the payoff from exercising is zero
  - A call options is out-of-the-money if $(S_t – X) < 0$ (i.e. if the stock price < the strike price)

Moneyness

- We say that an option is ***at-the-money*** if the price of the stock is equal to the strike price $(S_t=X)$

(i.e. the payoff is just about to turn positive)

- We say that an option is ***Deep-in-the-money*** if the payoff to exercise is extremely large
  - A call options is deep-in-the-money if $(S_t – X) > > 0$ (i.e. if the stock price > > the strike price)

A Put Option

A European* put option gives the buyer of the option ***a right to sell*** the underlying asset, at the contracted price (the ***exercise*** or ***strike price***) on a contracted future date (***expiration***)

*An ***American*** put option gives the buyer of the option (long put) a right to sell the underlying asset, at the exercise price, ***on or before*** the expiration date

Put Option - an Example

A March (European) put option on Microsoft stock with a strike price $20, entitles the owner with a right to sell the stock for $20 on expiration date.

What is the owner's payoff on expiration date? Under what circumstances does he benefit from the position?

The Payoff of a Put Option

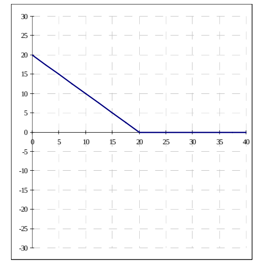
- On the expiration date:
  - If Microsoft stock was selling above \$20, the put would have been left to expire worthless.
  - If Microsoft had fallen below \$20, the put holder would have found it optimal to exercise.
- Exercise of the put is optimal if the stock price is below the exercise price:
  - Payoff at expiration is the maximum of two:

    Max {Exercise price - Stock price, 0} = Max {X - S_T, 0}

  - Profit at expiration = Payoff at expiration - Premium

Buying a Put –Payoff Diagram

Stock price = S_T	Payoff = Max{X-S_T, 0}
0	20
5	15
10	10
15	5
20	0
25	0
30	0



Writing a Put Option

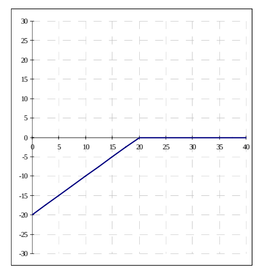
The seller of a put option is said to **write a put**, and he receives the option price called a **premium**. He is **obligated** to buy the underlying asset on expiration date (European), for the exercise price.

The payoff of a short put position (writing a put) is the negative of long put (buying a put):

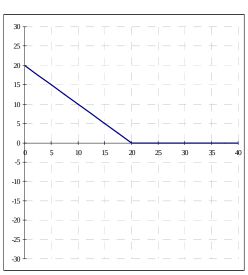
-Max {Exercise price - Stock price, 0} = -Max {X - S_T, 0}

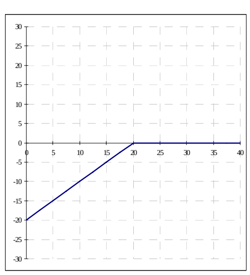
Writing a Put – Payoff Diagram

Stock price = S_T	Payoff = -Max{X-S_T,0}
0	-20
5	-15
10	-10
15	-5
20	0
25	0
30	0

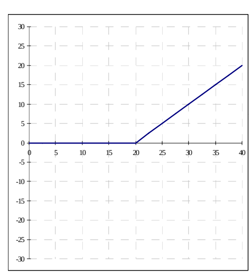


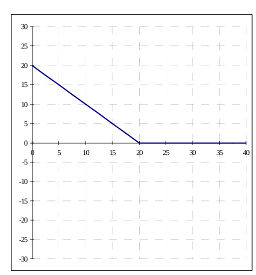
Buying a Put vs. Writing a Put Payoff Diagrams



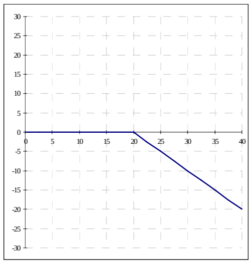


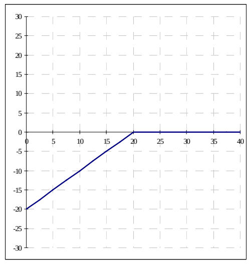
Buying a Call vs. Buying a Put Payoff Diagrams – Symmetry?





Writing a Call vs. Writing a Put Payoff Diagrams – Symmetry?



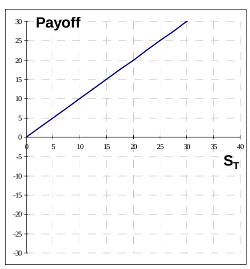


Long Position in a Stock

- ☺ The payoff increases as the value (price) of the stock increases
- ☺ The increase is one-for-one: for each dollar increase in the price of the stock, the value of the long position increases by one dollar

Long Stock – a Payoff Diagram

Stock price = S_T	Payoff = $+S_T$
0	0
5	5
10	10
15	15
20	20
25	25
30	30

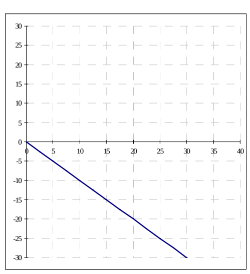


Short Position in a Stock

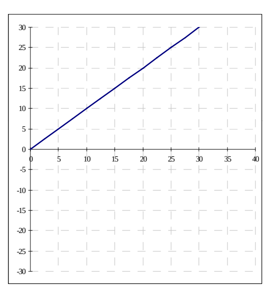
- ☺ The payoff decreases as the value (price) of the stock increases
- ☺ The decrease is one-for-one: for each dollar increase in the price of the stock, the value of the short position decreases by one dollar
- ☺ Note that the short position is a liability with a value equal to the price of the stock (mirror image of the long position)

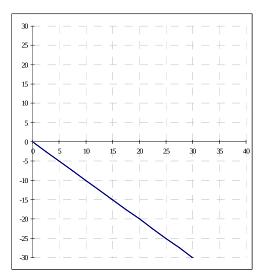
Short Stock – a Payoff Diagram

Stock price = S_T	Payoff = $-S_T$
0	0
5	-5
10	-10
15	-15
20	-20
25	-25
30	-30



Long vs. Short Position in a Stock – Payoff Diagrams



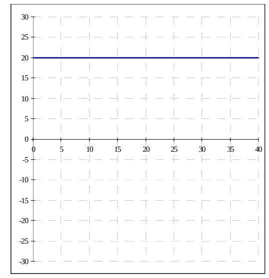


Long and Short Positions in the Risk-free Asset (Bond)

- ☺ The payoff is constant regardless of the changes in the stock price
- ☺ The payoff is positive for a lender (long bond) and negative for the borrower (short bond)

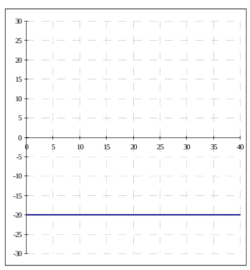
Lending – a Payoff Diagram

Stock price = S_T	Payoff = +X
0	20
5	20
10	20
15	20
20	20
25	20
30	20

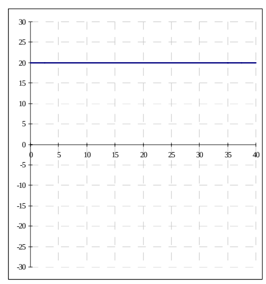


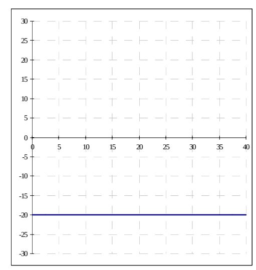
Borrowing – a Payoff Diagram

Stock price = S_T	Payoff = -X
0	-20
5	-20
10	-20
15	-20
20	-20
25	-20
30	-20



Lending vs. Borrowing Payoff Diagrams





Investment Strategies A Portfolio of Investment Vehicles

- ☺ We can use more than one investment vehicle to from a portfolio with the desired payoff.
- ☺ We can use any combination of the instruments (stock, bond, put or call) in any quantity or position (long or short) as our investment strategy.
- ☺ The payoff of the portfolio will be the sum of the payoffs of the instruments

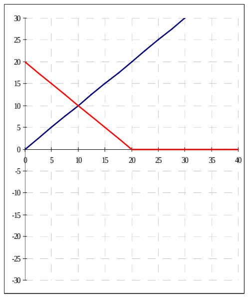
Investment Strategies: Protective Put

- ☺ Long one stock. The payoff at time T is: S_T
- ☺ Buy one (European) put option on the same stock, with a strike price of X = $20 and expiration at T. The payoff at time T is:

  Max { $X-S_T$, 0 } = Max { $20-$S_T$, 0 }
- ☺ The payoff of the portfolio at time T will be the sum of the payoffs of the two instruments
- ☺ Intuition: possible loses of the long stock position are bounded by the long put position

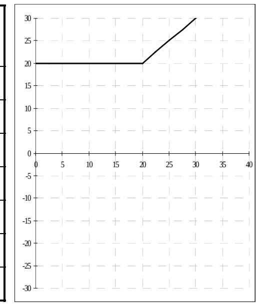
Protective Put – Individual Payoffs

Stock price	Long Stock	Buy Put
0	0	20
5	5	15
10	10	10
15	15	5
20	20	0
25	25	0
30	30	0



Protective Put – Portfolio Payoff

Stock price	Long Stock	Buy Put	All (Portfolio)
0	0	20	20
5	5	15	20
10	10	10	20
15	15	5	20
20	20	0	20
25	25	0	25
30	30	0	30



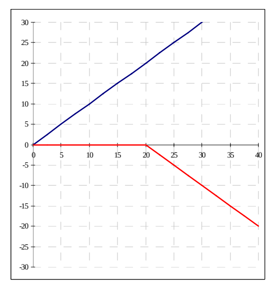
Investment Strategies Covered Call

- ☺ Long one stock. The payoff at time T is: S_T
- ☺ Write one (European) call option on the same stock, with a strike price of X = \$20 and expiration at T. The payoff at time T is:

  $-Max \{ S_T - X , 0 \} = -Max \{ S_T - \$20 , 0 \}$
- ☺ The payoff of the portfolio at time T will be the sum of the payoffs of the two instruments
- ☺ Intuition: the call is “covered” since, in case of delivery, the investor already owns the stock.

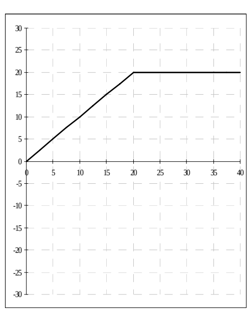
Covered Call – Individual Payoffs

Stock price	Long Stock	Write Call
0	0	0
5	5	0
10	10	0
15	15	0
20	20	0
25	25	-5
30	30	-10



Covered Call – Portfolio Payoff

Stock price	Long Stock	Write Call	All (Portfolio)
0	0	0	0
5	5	0	5
10	10	0	10
15	15	0	15
20	20	0	20
25	25	-5	20
30	30	-10	20



Other Investment Strategies

- ☺ Long straddle
  - Buy a call option (strike= X, expiration= T)
  - Buy a put option (strike= X, expiration= T)
- ☺ Write a straddle (short straddle)
  - Write a call option (strike= X, expiration= T)
  - Write a put option (strike= X, expiration= T)
- ☺ Bullish spread
  - Buy a call option (strike= X_1, expiration= T)
  - Write a Call option (strike= $X_2 > X_1$, expiration= T)

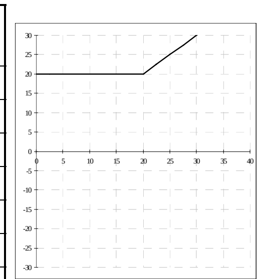
The Put Call Parity

Compare the payoffs of the following strategies:

- ☺ Strategy I:
  - Buy one call option (strike= X, expiration= T)
  - Buy one risk-free bond
    (face value= X, maturity= T, return= *rf*)
- ☺ Strategy II
  - Buy one share of stock
  - Buy one put option (strike= X, expiration= T)

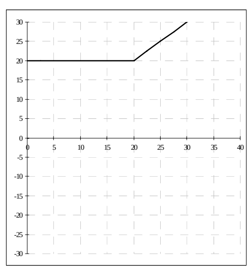
Strategy I – Portfolio Payoff

Stock price	Buy Call	Buy Bond	All (Portfolio)
0	0	20	20
5	0	20	20
10	0	20	20
15	0	20	20
20	0	20	20
25	5	20	25
30	10	20	30



Strategy II – Portfolio Payoff

Stock price	Buy Stock	Buy Put	All (Portfolio)
0	0	20	20
5	5	15	20
10	10	10	20
15	15	5	20
20	20	0	20
25	25	0	25
30	30	0	30



The Put Call Parity

If two portfolios have the same payoffs in every possible state and time in the future, their prices must be equal:

$$C + \frac{X}{(1+rf)^T} = S + P$$

Arbitrage – the Law of One Price

If two assets have the same payoffs in every possible state in the future but their prices are not equal, there is an opportunity to make an arbitrage profit.

We say that there exists an arbitrage profit opportunity if we identify that:

- There is no initial investment
- There is no risk of loss
- There is a positive probability of profit

Arbitrage – a Technical Definition

Let CF_{tj} be the cash flow of an investment strategy at time t and state j. If the following conditions are met this strategy generates an arbitrage profit.

(i) all the possible cash flows in every possible state and time are positive or zero - $CF_{tj} \geq 0$ for every t and j.

(ii) at least one cash flow is strictly positive - there exists a pair (t , j) for which $CF_{tj} > 0$.

Example

Is there an arbitrage profit opportunity if the following are the market prices of the assets:

The price of one share of stock is $39;

The price of a call option on that stock, which expires in one year and has an exercise price of $40, is $7.25;

The price of a put option on that stock, which expires in one year and has an exercise price of $40, is $6.50;

The annual risk free rate is 6%.

Example

In this case we should check whether the put call parity holds. Since we can see that this parity relation is violated, we will show that there is an arbitrage profit opportunity.

$$C + \frac{X}{(1+rf)^T} = \$7.25 + \frac{\$40}{(1+0.06)^1} = \$44.986$$

$$S + P = \$39 + \$6.50 = \$45.5$$

The Construction of an Arbitrage Transaction

Constructing the arbitrage strategy:

1. Move all the terms to one side of the equation so their sum will be positive;
2. For each asset, use the sign as an indicator of the appropriate investment in the asset. If the sign is negative then the cash flow at time t=0 is negative (which means that you buy the stock, bond or option). If the sign is positive reverse the position.

Example

In this case we move all terms to the LHS:

$$(S+P)-\left(C+\frac{X}{(1+rf)^T}\right)=\$45.5-\$44.986=\$0.514>0$$

i.e.

$$S+P-C-\frac{X}{(1+rf)^T}>0$$

Example

In this case we should:

1. Sell (short) one share of stock
2. Write one put option
3. Buy one call option
4. Buy a zero coupon risk-free bond (lend)

Example

Time: →	t = 0	t = T	
Strategy: ↓	State: →	$S_T < X = 40$	$S_T > X = 40$
Short stock			
Write put			
Buy call			
Buy bond			
Total CF	CF_0	CF_{T1}	CF_{T2}

Example

Time: →	t = 0	t = T	
Strategy: ↓	State: →	S_T < X = 40	S_T > X = 40
Short stock	+S=$39		
Write put	+P=$6.5		
Buy call	-C=(-$7.25)		
Buy bond	-X/(1+rf)=(-$37.736)		
Total CF	S+P-C-X/(1+rf) = 0.514		

Example

Time: →	t = 0	t = T	
Strategy: ↓	State: →	S_T < X = 40	S_T > X = 40
Short stock	+S=$39	$-S_T$	$-S_T$
Write put	+P=$6.5	$-(X-S_T)$	0
Buy call	-C=(-$7.25)	0	(S_T-X)
Buy bond	-X/(1+rf)=(-$37.736)	X	X
Total CF	S+P-C-X/(1+rf) = 0.514 > 0	$-S_T-(X-S_T)+X$ = 0	$-S_T-(X-S_T)+X$ = 0

Practice Problems

BKM 7th Ed. Ch. 20: 1-12, 14-23

BKM 8th Ed. Ch. 20:
5-14, 16-22, 26, CFA: 1-2

Practice Set: 1-16